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Gravitational Lorentz violations from M-theory

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Abstract: In an attempt to bridge the gap between M-theory and braneworld phenomenology, we present various gravitational Lorentz-violating braneworlds which arise from p-brane systems. Lorentz invariance is still preserved locally on the braneworld. For certain p-brane intersections, the massless graviton is *quasi*-localized. This also results from an M5-brane in a C-field. In the case of a p-brane perturbed from extremality, the quasi-localized graviton is massive. For a braneworld arising from *global* AdS_5, gravitons travel faster when further in the bulk, thereby apparently traversing distances faster than light.

Keywords: p-branes, Classical Theories of Gravity.

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Contents

1. Introduction

For certain non-factorizable five-dimensional geometries, four-dimensional gravity is recovered at low-energy scales [1, 2]. In particular, for a five-dimensional spacetime with a metric given by

$$ds_5^2 = a^2(z)(-dt^2 + dx_i^2 + dz^2)\,, \tag{1.1}$$

then for certain factors $a(z)$, gravity can be localized on a four-dimensional Minkowski surface of a domain wall with the directions t, x_i. For example, the second Randall-Sundrum braneworld scenario (RS2) used a warped five-dimensional metric that could be identified as a slice of AdS_5, for which $a(z) = (1 + k|z|)^{-1}$. The absolute value ensures $\mathbb{Z}_2$ symmetry about the domain wall, which corresponds to a delta-function potential well which localizes the massless graviton state.[1]

Additional factors in the metric (1.1) can yield two types of deviations from four-dimensional Minkowski space embedded in five dimensions. Firstly, additional factors could depend only on our four-dimensional spacetime, such as in the case of a cosmological spacetime on the braneworld. On the other hand, if additional factors depend only on the bulk dimension, then the spacetime on the braneworld will appear Minkowski for its inhabitants but Lorentz invariance will be broken globally [4]–[11]. The second case will be the focus of this paper. The corresponding metric can be written as[2]

$$ds_5^2 = a^2(z)(-b^2(z)\,dt^2 + dx_i^2 + dz^2)\,. \tag{1.2}$$

[1] Since an M-theoretic origin for this source is not known, a braneworld scenario which does not require this is proposed in [3].

[2] We do not include factors which would globally break rotational invariance, as this could be in contradiction with the isotropy of the cosmic microwave background [8].

Thus, the five-dimensional space is foliated by surfaces which exhibit Poincaré invariance for fixed z, leading to Lorentz-invariant Standard Model particle physics on the four-dimensional braneworld, located at $z = 0$. Gravitons, on the other hand, are not constrained to the braneworld. Therefore, gravitational processes may reflect the fact that four-dimensional Lorentz symmetry is broken globally.

In particular, from (1.2) the local speed of gravitational propagation[3] is given by $b(z)$. If $b(z)$ increases with z then gravitational effects may propagate from one region of our universe to another by bending in the extra dimension, thereby traveling faster than light on the braneworld.[4] Thus, regions of the universe not within each others' lightcones may actually affect each other via gravitation, leading to an apparent violation of causality. This suggested a non-inflationary solution to the cosmological horizon problem [4]–[6]. It is important to keep in mind that causality is still maintained from the five-dimensional vantage point.

Using Birkhoff's theorem, the metric (1.2) can always be transformed into that of a black hole, assuming the simplest possible sources in the bulk [8]. In particular, in the case of a Reissner-Nordstrom black hole in AdS_5, the graviton speed $b(z)$ increases away from the braneworld for a significant fraction of the charge-mass parameter space [8, 9, 11]. In a certain coordinate frame, the surface of the braneworld is bent in the bulk due to the presence of the black hole, while the graviton geodesics are straight lines. This is another way of visualizing that the gravitons would go into the bulk in order to connect two points which are on the braneworld.

We now ask the question: what M-theoretic factors give rise to a braneworld which is bent in the bulk? Although M-theory motivates the consideration of extra dimensions, it could be that braneworld scenarios are theoretically complete within a five-dimensional framework. However, certain aspects of braneworlds scenarios may have natural M-theoretic origins. Bridging the gap between M-theory and braneworld phenomenology may bring to light some connections with M-theoretic ideas, which may not be apparent from a five-dimensional vantage point. In this vein, we present various gravitational Lorentz-violating braneworld scenarios which arise naturally from p-brane systems. Although such scenarios generally arise from five-dimensional black holes, a plethora of higher-dimensional origins emerge within the framework of M-theory.

This paper is organized as follows. In section 2, we consider braneworld scenarios which lift to the near-horizon region of p-brane intersections. For the cases discussed, the spatial portion of the braneworld corresponds to relative transverse dimensions in the p-brane system, which results in Lorentz invariance being globally broken. In section 3, we find that a braneworld scenario resulting from the near-horizon region of an M5-brane in a constant C-field also exhibits the global breaking of Lorentz invariance. For both scenarios, the massless graviton is *quasi*-localized to the brane. In section 4, we present an example of a globally Lorentz-violating braneworld scenario arising from the near-horizon region of a p-brane which is perturbed from extremality, and show that the quasi-localized graviton

[3] One can rescale t in order for the speed of light on the braneworld at $z = 0$ to be unity.

[4] One recent proposal for measuring the speed of propagation of gravity (over relatively short distances) was given in [12].

JHEP10(2002)053

is massive. In section 5, we consider a braneworld scenario which arises from AdS_5, which could be interpreted as the near-horizon region of a D3-brane reduced on S^5. For *global* AdS, the breaking of global Lorentz invariance leads to gravitons on the braneworld which, from our vantage point, traverse distances faster than light. In section 6, we discuss some open issues.

2. Braneworld from intersecting p-branes

Albeit the imposed $\mathbb{Z}_2$ symmetry about $z = 0$, five-dimensional gravity-trapping domain walls arise from ten dimensions as sphere reductions of the near-horizon region of Dp-branes for $p = 3, 4, 5$. The domain wall resulting from a D3-brane was originally used in the RS2 braneworld scenario while the D4 and D5 branes yield dilatonic domain walls. In addition, the D4 and M5 branes dimensionally reduce to the same domain wall, as does the D5-brane and M5/M5-brane intersection [13].

In the aforementioned cases, the observed four-dimensional spacetime lies entirely within the (overall) worldvolume of the p-brane(s). This is partially motivated by holography, since the above cases for which gravity is localized all have a natural decoupling limit, which suggests that the localization of gravity may generally be realized within a Domain-wall/QFT correspondence.[5] For example, in the original RS2 scenario, the directions of the effective four-dimensional universe correspond to the worldvolume of a D3-brane in ten dimensions. If the worldvolume of the D3-brane is Minkowski, then so is that of the braneworld, thus yielding an effective four-dimensional universe that is strictly Lorentz invariant. However, one could envision a system of intersecting p-branes for which some of the resulting braneworld directions do not lie within the overall worldvolume of the p-branes. If all of the three observed spatial dimensions are relative transverse to a p-brane, the result may be a braneworld scenario which globally breaks gravitational Lorentz invariance, while spatial rotational invariance is maintained.

2.1 Braneworld from NS1/D2 system

One way to achieve this is to add a pp-wave along the worldvolume of a D3-brane in type IIB theory. The near-horizon limit of the solution then becomes $K_5 \times S^5$, where K_5 is the generalized Kaigorodov metric in $D = 5$. While this supergravity solution is dual to a conformal field theory in the infinite momentum frame [14], this interpretation is lost once we T-dualize along the direction of the pp-wave. The resulting type IIA solution is an NS1/D2 system of type IIA theory. The corresponding metric is given by

$$ds_{10}^2 = H_1^{-3/4} H_2^{-5/8} \left(-dt^2 + H_2 dx_1^2 + H_1(dx_2^2 + dx_3^2) + H_1 H_2 (dr^2 + r^2 d\Omega_5^2)\right), \qquad (2.1)$$

where

$$H_1 = 1 + \frac{R_1^4}{r^4}, \qquad H_2 = 1 + \frac{R_2^4}{r^4}, \qquad (2.2)$$

and R_1 and R_2 are the charges of the NS1 and D2 brane, respectively. This brane system can be illustrated by the diagram 1.

[5] The delta-function domain-wall source due to the imposed $\mathbb{Z}_2$ symmetry provides an ultra-violet cut-off in a dual quantum field theory.

JHEP10(2002)053

	t	x_1	x_2	x_3	r	s_1	s_2	s_3	s_4	s_5	
D2	$\times$	$-$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	H_2
NS1	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	H_1

Table 1: The NS1/D2 system.

In the near-horizon region, we neglect the "1" in the harmonic functions and, for simplicity, consider the case $R_1 = R_2 \equiv R$. The metric (2.1) can then be expressed as

$$ds_{10}^2 = (1 + k|z|)^{-3/2} \left(-(1 + k|z|)^{-4} dt^2 + dx_i^2 + dz^2\right) + (1 + k|z|)^{1/2} R^2 d\Omega_5^2 \,, \tag{2.3}$$

where

$$\frac{r}{R} = (1 + k|z|)^{-1} \,, \tag{2.4}$$

and $i = 1, 2, 3$. We use the following metric Ansatz for dimensional reduction over S^n

$$ds_D^2 = \mathrm{e}^{-2\alpha\phi} ds_d^2 + g^{-2} \mathrm{e}^{(2(d-2))/n\alpha\phi} d\Omega_n^2 \,, \tag{2.5}$$

where $\alpha = -\sqrt{n/(2(D-2)(d-2))}$. Reducing over S^5 in (2.3) using the Ansatz (2.5) we obtain

$$ds_5^2 = (1 + k|z|)^{-2/3} \left(-(1 + k|z|)^{-4} dt^2 + dx_i^2 + dz^2\right) . \tag{2.6}$$

The fluctuations of the five-dimensional graviton satisfy the equation of a minimally-coupled scalar field in a gravitational background given by

$$\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \Phi = 0 \,. \tag{2.7}$$

Taking $\Phi = M(t, x_i)\,(1 + k|z|)^{3/2} \psi(z)$ we obtain a Schrödinger-type wave equation,

$$-\partial_z^2 \psi + V(z)\psi = m^2 \psi \,, \tag{2.8}$$

with an effective potential given by

$$V(z) = \frac{15k^2}{4(1 + k|z|)^2} - 3\delta(z) \,. \tag{2.9}$$

Note that the mass m is defined by

$$\Box_{(4)} M(t, x_i) = m^2 M(t, x_i) \,, \tag{2.10}$$

where $\Box_{(4)}$ is the four-dimensional laplacian.

The graviton wave equation (2.8) is the same as for a braneworld originating from the D3-brane near-horizon region, such as the original RS2 scenario. The massless graviton is localized on the braneworld, as can by seen by the fact that the corresponding wavefunction,

$$\psi = N(1 + k|z|)^{-3/2} \,, \tag{2.11}$$

is square-normalizable. The effective four-dimensional newtonian potential is modified in the same way, via the massive graviton modes, as for braneworld arising from a D3-brane.

	t	x_1	x_2	x_3	x_4	r	s_1	s_2	s_3	s_4	
D2	$\times$	$-$	$-$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	H
D2	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	K

Table 2: The D2/D2 system.

In the present case, however, Lorentz invariance is globally broken, as can be seen from the form of the metric (2.6). The speed of gravitons in the bulk is $v(z) = (1 + k|z|)^{-2}$, where the speed of light on the braneworld is unity. Since $v(z)$ decreases away from the braneworld, gravitons will not appear to travel faster than the speed of light. However, a Lorentz-violating effect, from the point of view of the braneworld, is a modification of the dispersion relation, $\omega^2 = m^2 + c^2\,\vec{p}^2$, for which c now depends on the spread of the wave function in the extra dimension [8], [15]–[17].

The metric (2.6) is of the form (1.2) where $b(z) \longrightarrow 0$ as $|z| \longrightarrow \infty$. In addition, $V(z)$ given by (2.9) tends to a constant for large $|z|$. This implies that the massless graviton is now *quasi*-localized for $\vec{p}^{\,2} > 0$, that is, metastable against escape into the extra dimension. In particular, the escape width $\Gamma(|\vec{p}|)$ increases with $|\vec{p}|$ [18]. This is true also for massless particles other than the graviton, provided that the corresponding fields have bulk modes. This type of metastability may have phenomenological consequences, especially for ultra-high energy cosmic rays [15, 19, 20].

2.2 Other examples

We find similar results if we add a pp-wave to the worldvolume of a D4 or D5-brane and then T-dualize the solution. In the case of a D4-brane with a pp-wave, T-dualizing along the direction of the wave yields the NS1/D3 system of type IIB theory. Note that this is dual to the type IIA D2/D2 system, by lifting up to eleven dimensions and the reducing along a different direction. The D2/D2 system can be represented by the diagram 2.

Without going into the details, in the near-horizon region we reduce over the transverse S^4 and one of the x_i directions by using the metric Ansatz (2.5). After a coordinate transformation, we obtain

$$ds_5^2 = (1 + k|z|)^{-4/3}\left(-(1 + k|z|)^{-6}dt^2 + dx_i^2 + dz^2\right). \tag{2.12}$$

Taking $\Phi = M(t, x_i)(1 + k|z|)^{3/2}\psi(z)$ yields a Schrödinger-type wave equation,

$$\partial_z^2\psi + \left(\frac{35k^2}{4(1 + k|z|)^2} - 5\delta(z)\right)\psi = m^2\psi\,. \tag{2.13}$$

This is the identical graviton wave equation as for the braneworld resulting from the D4-brane.

In a similar manner, T-dualizing a D5-brane with a pp-wave yields the NS1/D4, which is dual to the D3/D3 system. This brane system can be illustrated by the diagram 3.

In the near-horizon region, a reduction over the transverse S^3 and two of the x_i directions yields

$$ds_5^2 = \mathrm{e}^{-\frac{k}{3}|z|}(-\mathrm{e}^{-k|z|}dt^2 + dx_i^2 + dz^2)\,, \tag{2.14}$$

	t	x_1	x_2	x_3	x_4	x_5	r	s_1	s_2	s_3	
D3	×	×	−	−	×	×	−	−	−	−	H
D3	×	×	×	×	−	−	−	−	−	−	K

Table 3: The D3/D3 system.

after a coordinate transformation. Taking $\Phi = M(t,x_i)(1+k|z|)^{3/2}\psi(z)$, we obtain a Schrödinger-type wave equation

$$\partial_z^2\psi + \left(\frac{k^2}{4} - \delta(z)\right)\psi = m^2\psi\,. \tag{2.15}$$

This is the same graviton wave equation as for the braneworld from the D5-brane.

In all of these scenarios, the graviton speed decreases away from the braneworld, so we would not observe causality violations induced by bulk gravitational propagation. However, as previously mentioned, the dispersion relation is modified and the massless graviton is quasi-localized.

3. Braneworld from M5-brane with C-field

Consider the M5-brane solution with a constant C-field [21]:

$$\begin{aligned}
ds_{11}^2 &= K^{1/3}H^{1/3}\left(\frac{1}{H}(-dt^2+dx_4^2+dx_5^2)+\frac{1}{K}dx_i^2+dr^2+r^2d\Omega_4^2\right),\\
H &= 1+\frac{R^3}{r^3}\,, \qquad K=\sin^2\theta+\cos^2\theta\,H\,,\\
dC_{(3)} &= \sin\theta\,dH^{-1}\wedge dt\wedge dx_4\wedge dx_5+3R^3\Omega_{(4)}\cos\theta-6\tan\theta\,dK^{-1}\wedge dx_1\wedge dx_2\wedge dx_3\,,
\end{aligned}$$

where $i=1,2,3$. This can be interpreted as a 2-brane lying within a 5-brane [22]. In the near-horizon region we can neglect the "1" in H. Reducing over S^4, x_4 and x_5 yields a five-dimensional domain wall with the metric

$$ds_5^2 = (1+k|z|)^{-10/3}\left([\cos^2\theta+\sin^2\theta\,(1+k|z|)^{-6}](-dt^2+dz^2)+dx_i^2\right), \tag{3.1}$$

where $r/R=(1+k|z|)^{-2}$. Taking $\Phi = M(t,x_i)(1+k|z|)^{5/2}\psi(z)$, we find that the graviton equation of motion can be expressed as

$$-\partial_z^2\psi + \left(\frac{35k^2}{4(1+k|z|)^2} - 5k\,\delta(z)\right)\psi = m^2\left(\cos^2\theta + \frac{sin^2\theta}{(1+k|z|)^6}\right)\psi\,. \tag{3.2}$$

The massless case coincides with the braneworld scenario arising from the near-horizon of a D4-brane or M5-brane without a C-field, given by (2.13) and studied in [13]. The graviton speed decreases away from the braneworld which, as with the p-brane intersections discussed in the previous section, results in a modification of the dispersion relation and a *quasi*-localized massless graviton. Since these braneworlds are bent in the bulk, graviton geodesics may still connect two points on the braneworld by protruding into the bulk. As a result of this, from the perspective of the dual gauge theory, the background metric is smeared out over a typical length scale R [23]. This seems to be consistent with the expectation that the decoupling limit of the M5-brane with a constant C-field has a dual gauge theory which is non-associative.

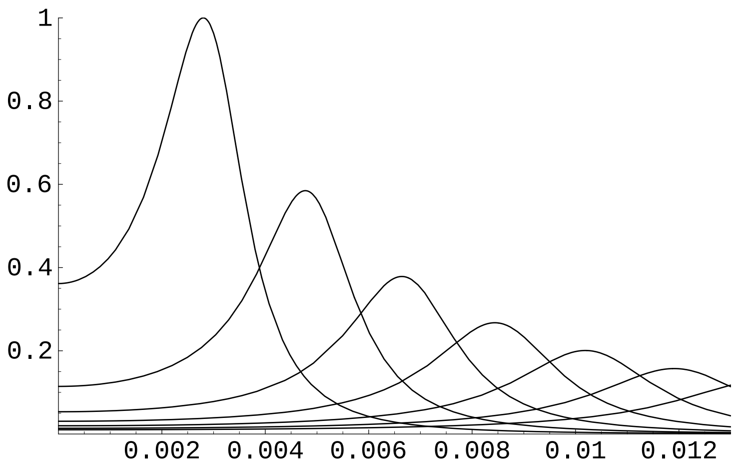

Figure 1: $|\psi(z=0)|^2$ versus m/k for $e = .005$, $.01$, $.015$, $.02$, $.025$, $.03$, and $.035$.

4. Braneworld from non-extremal p-brane

The gravitational breaking of Lorentz invariance may correspond to a perturbation away from extremality in the higher-dimensional p-brane origin of the braneworld scenario, which corresponds to a finite temperature in the dual gauge theory. A case that is tractable with semi-analytical methods [24] is the 5-brane solution of heterotic or type II theories, whose metric is given by

$$ds_{10}^2 = H^{-1/4}(-f\,dt^2 + dx_j^2) + H^{3/4}(f^{-1}dr^2 + r^2 d\Omega_3^2)\,, \tag{4.1}$$

where

$$H = 1 + \frac{R^2}{r^2}\,, \quad f = 1 - \frac{e^2 R^2}{r^2}\,, \tag{4.2}$$

and $j = 1, \ldots, 5$. The non-extremality parameter is e. We suppose that the brane is perturbed slightly from extremality so that $e \ll 1$. In the near-horizon region we can neglect the “1” in H, and the metric (4.1) can be expressed as

$$ds_{10}^2 = (\mathrm{e}^{-k|z|} + e^2)^{1/4}\left(f(-dt^2 + dz^2) + dx_j^2 + R^2 d\Omega_3^2\right), \tag{4.3}$$

where $r/R = \sqrt{\mathrm{e}^{-k|z|} + e^2}$. Reducing this region of the 5-brane metric on $T^2 \times S^3$ with the Ansatz (2.5) yields a five-dimensional domain wall with the metric

$$ds_5^2 = (\mathrm{e}^{-k|z|} + e^2)^{2/3}\left((1 + e^2\,\mathrm{e}^{k|z|})^{-1}(-dt^2 + dz^2) + dx_i^2\right), \tag{4.4}$$

where $i = 1, 2, 3$. The speed of gravitons in the bulk is $v(z) = (1 + e^2\,\mathrm{e}^{k|z|})^{-1}$. Since $v(z)$ decreases away from the braneworld, gravitons will not appear to travel faster than the speed of light. However, as we shall see, one consequence of the global breaking of Lorentz invariance is that the effective four-dimensional graviton is now a quasi-localized massive state.

Considering a graviton fluctuation which obeys the equation of motion for a minimally-coupled scalar given by (2.7), we take $\Phi = (\mathrm{e}^{-k|z|} + e^2)^{-1/2} M(t, x_i)\psi(z)$. The resulting wave equation is

$$-\partial_z^2 \psi + \left(\frac{k^2/2 - m^2}{1 + e^2 \mathrm{e}^{k|z|}} - \frac{k^2}{4(1 + e^2 \mathrm{e}^{k|z|})^2} - \frac{k}{1 + e^2} \delta(z) \right) \psi = 0 . \tag{4.5}$$

Note that m^2 is the eigenvalue of the four-dimensional laplacian acting on $M(t, x_i)$. For the massless mode, this reduces to a Schrödinger-type wave equation with the solution given by

$$\psi = N(e^2 + \mathrm{e}^{-k|z|})^{1/2} . \tag{4.6}$$

For the extremal case, this solution is square-normalizable and the massless graviton is therefore bound to the braneworld [13]. Away from extremality, the solution diverges for large z and is no longer physical. However, there is a quasi-localized massive graviton mode, whose mass increases away from extremality. This is illustrated numerically in figure 1, by a massive resonance on the braneworld. Notice that the wave function dissipates further away from the extremal case, which implies a decrease in the distance that the massive graviton propagates on the braneworld before escaping into the extra dimension.

The non-zero mass of the quasi-bound graviton implies that it will travel with a speed less than that of light on the brane. However, the lower bound on graviton speed is tightly constrained by high-energy cosmic ray observations [25]. On the other hand, since the braneworld is bent in the bulk, graviton geodesics can still connect two points on the braneworld by protruding into the bulk. This could lead to a smeared out background metric in the dual gauge theory [23], which might be connected with the finite temperature.

5. Braneworld from global AdS_5

The RS2 braneworld model used a warped five-dimensional metric that could be identified as a slice of AdS_5. Albeit the imposed $\mathbb{Z}_2$ symmetry, this can be obtained from the near-horizon region of the D3-brane reduced over S^5.

Consider the metric for AdS_5 expressed in global coordinates, which is given by

$$ds_5^2 = -\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \, dx_i^2 . \tag{5.1}$$

Imposing $\mathbb{Z}_2$ symmetry about z we can express (5.1) as

$$ds_5^2 = \frac{1}{\sinh^2(1 + k|z|)} (-\cosh^2(1 + k|z|) dt^2 + dx_i^2 + dz^2) , \tag{5.2}$$

where $\sinh\rho = 1/\sinh(1 + k|z|)$.

We shall show that there is a massless graviton mode localized on the braneworld. We insert the graviton wavefunction $\Phi = \phi(z) M(t, x_i)$ into (2.7) for the background given by (5.2) and find the radial wave equation to be

$$-\frac{\sinh^3(1 + k|z|)}{\cosh(1 + k|z|)} \partial_z \frac{\cosh(1 + k|z|)}{\sinh^3(1 + k|z|)} \partial_z \phi = m^2 \phi , \tag{5.3}$$

where m is defined as in the previous sections. With the wave function transformation

$$\phi = \left(\frac{\cosh(1+k|z|)}{\sinh^3(1+k|z|)} \right)^{-1/2} \psi \,, \tag{5.4}$$

the wave equation (5.3) can be expressed in Schrödinger form,

$$-\partial_z^2 \psi + V(z)\psi = m^2 \psi \,, \tag{5.5}$$

with

$$V(z) = \frac{4\sinh^4(1+k|z|) + 20\sinh^2(1+k|z|) + 15}{\sinh^2(2(1+k|z|))} k^2 - \alpha\, k\, \delta(z) \,, \tag{5.6}$$

where $\alpha = 2(2\sinh^2(1)+3)/\sinh(2)$. This is a volcano-type potential. That is, the coefficient of the delta-function term is negative, and the k^2 term is maximum at $z=0$ and decreases to $V(z \to \infty) = k^2$. The latter property indicates that there is a mass gap of $M_{\rm gap} = k$ separating the localized massless state from the massive modes which propagate in the extra dimension.[6]

The massless wavefunction solution is given by

$$\psi = N \sqrt{\frac{\cosh(1+k|z|)}{\sinh^3(1+k|z|)}} \,, \tag{5.7}$$

where N is the normalization constant. Since this wavefunction is square normalizable, it corresponds to a localized massless graviton state.

In order to see if the original RS2 scenario can arise within a certain limit of the present braneworld scenario, we re-express the metric for global AdS_5 (5.1) via the coordinate transformation $\sinh\rho = 1/\sinh(\gamma + k|z|)$. Rather than setting $\gamma = 1$, we consider the case where γ is a small constant. In the limit $\gamma + k|z| \ll 1$, the geometry asymptotes to a local slice of AdS_5. Therefore, in the vicinity of our braneworld, we expect the physics to be similar to that of the original RS2 scenario. Indeed, in this limit with γ replacing the "1" in the massless graviton wavefunction, (5.7) reduces to $\psi = N(\alpha + k|z|)^{-3/2}$, which is the massless wavefunction for the RS2 scenario for this particular choice of coordinates.

Further away from the braneworld, the present scenario differs quite drastically from the RS2 scenario. This is demonstrated by the speed of gravitons in the bulk, which is $v(z) = \cosh(1+k|z|)/\cosh(1)$, after a rescaling of t. Since $v(z)$ increases away from the braneworld, there is an apparent violation of causality. That is, as discussed in the Introduction, gravitational disturbances may bend into the bulk and arrive at a particular location on the braneworld earlier than does the light from the same source.

It would seem that this implies the presence of a tachyon in the dual gauge theory, which is inherently four-dimensional [26]. However, in the present case, the dual gauge theory is in a stable phase.

[6]Note that (5.6) is the same graviton wave equation as for D3-branes distributed uniformly over a disc [27].

6. Discussion

We have found that the breaking of global Lorentz invariance for a braneworld may stem from various M-theoretic causes. One possibility is that the spatial dimensions of a braneworld correspond to relative transverse dimensions in the near-horizon region of a system of intersecting p-branes. Another higher-dimensional origin is an M5-brane in a constant C-field. For such braneworld scenarios, the massless graviton, and any other massless particle with corresponding bulk modes, is *quasi*-localized with an escape width which increases with momentum. Such metastability may have phenomenological consequences, for example, for ultra-high energy cosmic rays. As we discussed, another consequence in such a scenario is a modification of the dispersion relation.

We next discussed the braneworld scenario which results from the near-horizon region of a p-brane that is perturbed away from extremality. In particular, we considered the semi-analytically tractable example of a 5-brane in heterotic or type II theories. The global breaking of Lorentz invariance in the corresponding braneworld gives rise to a *quasi*-localized graviton which is massive. As the corresponding 5-brane is perturbed further from extremality, the mass of the quasi-bound state increases, and the graviton propagates for a decreasing distance on the braneworld before escaping into the bulk. Another odd consequence, which was discussed in [24], is the presence of quasi-bound massive harmonic states, which travel a relatively short distance on the brane before escaping.

We considered a braneworld scenario which arises from global AdS_5. Unlike the case of a local patch of AdS_5, which was used in the original RS2 scenario, Lorentz invariance is globally broken on the brane since the speed of gravitons increases away from the braneworld. As discussed in the Introduction, the result of this is that gravitons can travel from one point of the braneworld to another by bending into the bulk, and thus arriving apparently faster than light.

It seems that, in order to take the Penrose limit of $AdS_5 \times S^5$, AdS must be expressed in global coordinates.[7] Therefore, there may be an interesting connection with the braneworld scenario arising from $AdS_5 \times S^5$, for which AdS is in global coordinates. In taking this limit, one considers particles traveling fast around a coordinate in S^5. Therefore, one would need to consider gravitational modes which move fast around S^5 [32].

The aforementioned Lorentz violations can be transmitted to the Standard Model sector via gravitational loops. However, as the energy scale increases, gravity on the braneworld does not probe as far into the bulk. That is, at short distances the speed of gravitons approaches the speed of light on the brane. Therefore, gravitational loops which transmit Lorentz violations to non-gravitational particles are suppressed at the energy scale where gravity becomes strongly interacting [8].

Lastly, since this falls under the category of gravitational Lorentz violations from M-theory, we would like to make mention that five-dimensional AdS-Reissner-Nordstrom black holes have the ten-dimensional interpretation of spinning D3-branes [33]. This enables us to place previous work on gravitational Lorentz violations within the context of ten dimensions

[7] This results in a background on which the type IIB light-cone string action is exactly solvable [28]–[31].

JHEP10(2002)053

and holography, in particular with regards to the finite-density Coulomb branch of the dual gauge theory. This point is elaborated in [34].

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References

[1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[2] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[3] J.F. Vázquez-Poritz, *Gravity-trapping domain walls from resolved branes*, *J. High Energy Phys.* **09** (2002) 001 [hep-th/0111229].

[4] G. Kalbermann, *Communication through an extra dimension*, *Int. J. Mod. Phys.* **A 15** (2000) 3197 [gr-qc/9910063].

[5] D.J.H. Chung and K. Freese, *Can geodesics in extra dimensions solve the cosmological horizon problem?*, *Phys. Rev.* **D 62** (2000) 063513 [hep-ph/9910235].

[6] H. Ishihara, *Causality of the brane universe*, *Phys. Rev. Lett.* **86** (2001) 381 [gr-qc/0007070].

[7] D.J.H. Chung, E.W. Kolb and A. Riotto, *Extra dimensions present a new flatness problem*, *Phys. Rev.* **D 65** (2002) 083516 [hep-ph/0008126].

[8] C. Csáki, J. Erlich and C. Grojean, *Gravitational Lorentz violations and adjustment of the cosmological constant in asymmetrically warped spacetimes*, *Nucl. Phys.* **B 604** (2001) 312 [hep-th/0012143].

[9] C. Csáki, J. Erlich and C. Grojean, *The cosmological constant problem in brane-worlds and gravitational Lorentz violations*, *Gen. Rel. Grav.* **33** (2001) 1921 [gr-qc/0105114].

[10] C. Grojean, F. Quevedo, G. Tasinato and I. Zavala C., *Branes on charged dilatonic backgrounds: self-tuning, Lorentz violations and cosmology*, *J. High Energy Phys.* **08** (2001) 005 [hep-th/0106120].

[11] C. Csáki, *Asymmetrically warped spacetimes*, hep-th/0110269.

[12] S. Kopeikin and E. Fomalont, *General relativistic model for experimental measurement of the speed of propagation of gravity by VLBI*, gr-qc/0206022.

[13] M. Cvetič, H. Lü and C.N. Pope, *Domain walls with localised gravity and domain-wall/QFT correspondence*, *Phys. Rev.* **D 63** (2001) 086004 [hep-th/0007209].

[14] M. Cvetič, H. Lü and C.N. Pope, *Spacetimes of boosted p-branes and CFT in infinite-momentum frame*, *Nucl. Phys.* **B 545** (1999) 309 [hep-th/9810123].

[15] V.A. Rubakov, *Large and infinite extra dimensions: an introduction*, *Phys. Usp.* **44** (2001) 871 [hep-ph/0104152].

[16] S.R. Coleman and S.L. Glashow, *High-energy tests of Lorentz invariance*, *Phys. Rev.* **D 59** (1999) 116008 [hep-ph/9812418].

[17] C.P. Burgess, J. Cline, E. Filotas, J. Matias and G.D. Moore, *Loop-generated bounds on changes to the graviton dispersion relation*, *J. High Energy Phys.* **03** (2002) 043 [hep-ph/0201082].

[18] S.L. Dubovsky, *Tunneling into extra dimension and high-energy violation of Lorentz invariance*, *J. High Energy Phys.* **01** (2002) 012 [hep-th/0103205].

[19] O. Bertolami and C.S. Carvalho, *Proposed astrophysical test of Lorentz invariance*, *Phys. Rev.* **D 61** (2000) 103002 [gr-qc/9912117].

[20] O. Bertolami, *Ultra-high energy cosmic rays and symmetries of spacetime*, *Gen. Rel. Grav.* **34** (2002) 707.

[21] J.M. Maldacena and J.G. Russo, *Large-N limit of non-commutative gauge theories*, *J. High Energy Phys.* **09** (1999) 025 [hep-th/9908134].

[22] J.G. Russo and A.A. Tseytlin, *Waves, boosted branes and BPS states in M-theory*, *Nucl. Phys.* **B 490** (1997) 121 [hep-th/9611047].

[23] P. Creminelli, *Holography of asymmetrically warped space-times*, *Phys. Lett.* **B 532** (2002) 284 [hep-th/0111107].

[24] J.F. Vázquez-Poritz, *Massive gravity on a non-extremal brane*, *J. High Energy Phys.* **12** (2001) 030 [hep-th/0110299].

[25] G.D. Moore and A.E. Nelson, *Lower bound on the propagation speed of gravity from gravitational cherenkov radiation*, *J. High Energy Phys.* **09** (2001) 023 [hep-ph/0106220].

[26] P. Creminelli, private correspondence.

[27] A. Brandhuber and K. Sfetsos, *Non-standard compactifications with mass gaps and Newton's law*, *J. High Energy Phys.* **10** (1999) 013 [hep-th/9908116].

[28] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, *J. High Energy Phys.* **01** (2002) 047 [hep-th/0110242].

[29] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. and Quant. Grav.* **19** (2002) L87 [hep-th/0201081].

[30] R.R. Metsaev, *Type-IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[31] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[32] J.F. Vázquez-Poritz, *Rotational graviton modes in the brane world*, *J. High Energy Phys.* **07** (2002) 019 [hep-th/0202029].

[33] M. Cvetič, M.J. Duff, P. Hoxha, J.T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati and T.A. Tran, *Embedding AdS black holes in ten and eleven dimensions*, *Nucl. Phys.* **B 558** (1999) 96 [hep-th/9903214].

[34] J.F. Vázquez-Poritz, *Phases of braneworlds, spinning D3-branes and strongly-coupled gauge theories*, hep-th/0209194.

JHEP10(2002)053